1. One item costs d dollars, and a second item costs 2d dollars. I get a 4% discount on the first item and a 10% discount on the second. What is my overall discount?

   A. 5% B. 6% C. 7% D. 8% E. 9%

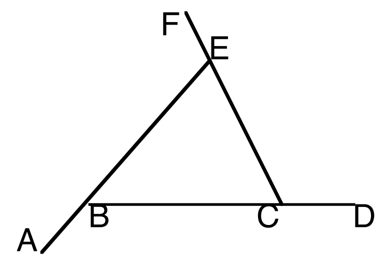
2. Jo buys 2 gal of gas and 2 qt of oil. A gallon of gas costs $1.20 more than a quart of oil; the price per qt of oil is 60% of the price per gal of gas. How much does Jo pay?

   A. $8.40 B. $9.00 C. $9.60 D. $10.20 E. $10.80

3. Given ΔBCE in the figure at the right, if m∠ABC = x°, m∠DCE = (x + 10)°, and m∠BEF = (x - 15)°, in which interval below does the value of x lie?

   A. [114, 117) B. [117, 120) C. [120, 123)

   D. [123, 126) E. [126, 129)



4. Ed has an equal number of dimes and nickels and no pennies. After spending 15¢, he has twice as many dimes as nickels. How many nickels did he have originally?

   A. 3 B. 4 C. 5 D. 6 E. 7

5. Let A and B be single nonzero digits, so that AA is a 2-digit number with identical digits. Which of the following is a value of A which satisfies AA + A = B * A^B?

   A. 1 B. 2 C. 3 D. 4 E. 5

6. Each member of a group of 5 people is either a knight, who always tells the truth, or a knave, who always lies. Each member of the group looks at everyone else in the group, and then one member says, "I see at least one person who sees only knaves." What is the least possible number of knights in the group?

   A. 0 B. 1 C. 2 D. 3 E. 4

7. There are exactly 2 noncongruent rectangular boxes with integer-length edges whose space diagonals have length 17. Find the ratio of the larger to the smaller volume.

   A. 2 B. 3 C. 4 D. 5 E. 6

8. A positive integer N leaves the remainder r ≠ 0 when divided into any of 527, 622, and 698. A different positive integer M leaves the remainder s ≠ 0 when divided into any of 736, 881, and 997. Find M + N.

   A. 46 B. 48 C. 50 D. 52 E. 54

9. The three-digit number abc is 29 times the sum of its digits, the number bca is 68 times the sum of its digits, and the number cab is 14 times the sum of its digits. Find a + 2b + 3c. A. 14 B. 15 C. 17 D. 20 E. 23

10. In ΔSML, SM = 8, ML = 12, and LS = 18. Of the 3 sides and 3 angles of ΔSML, five of these parts are congruent to five parts of ΔTYC, but ΔSML is NOT congruent to ΔTYC. If the sides of ΔTYC are all integers, find the longest side of ΔTYC.

    A. 18 B. 24 C. 27 D. 30 E. 36

11. Every integer $N > 0$ can be represented in at least one way as $ab - (a + b)$, where a and b are positive integers with $a \le b$. Find the least N having at least 3 such representations, and write your answer on the answer sheet.

12. What is the remainder when $x^{2000} - 2x^{15} + 2$ is divided by $x^2 - 1$?

    A. $-2x + 3$ B. $2x - 3$ C. $2x + 3$ D. $-2x - 3$ E. 3

13. Find all values of k such that the equation $\log(kx) = 2\log(x + 1)$ has exactly one real solution.

    A. $k = 4$ B. $k < 0$ C. $k = -1$ D. A or B E. A or C

14. In the system of equations $\begin{cases} xy + z = 105 \\ x + yz = 107 \end{cases}$, for how many integer values of y does the system have a solution in integers? A. 0 B. 1 C. 2 D. 3 E. 4

15. In ΔABC, $\angle A = 30°$, $\angle C = 90°$, and $AB = 2\sqrt{6}$. Let D be the point on side AB such that segment CD bisects $\angle C$. Find CD.

    A. $3\sqrt{3} - 3$ B. $4\sqrt{2} - 3$ C. $2\sqrt{6} - 2$ D. $2\sqrt{2}$ E. $3\sqrt{2} - 2$

16. If $\sin x \cos x = 4 \sin x + 4 \cos x$, $\sin 2x$ can be represented in the form $a + b\sqrt{c}$, where a, b, and c are integers and c is a prime number. Find $a + b + c$.

    A. 41 B. 45 C. 49 D. 53 E. 57

17. A circle with center O has radius 10 and is tangent to $\angle BAC$ at points B and C. A circle with center Q is tangent to the first circle at P and to $\angle BAC$ at D and E. A circle with center S is tangent to the second circle at R and to $\angle BAC$ at F and G. If $OP > QR > RS$ and $OA = 25$, find $\sqrt{AS}$.

    A. $\frac{13}{7}$ B. $\frac{15}{7}$ C. $\frac{16}{7}$ D. $\frac{18}{7}$ E. $\frac{19}{7}$

18. Let $a + b = M$ and $ab = N$. The polynomial $a^4 + b^4$ can be represented as $PM^4 - QM^2N + RN^2$, where P, Q and R are positive integers. Find $P + Q + R$.

    A. 4 B. 5 C. 6 D. 7 E. 8

19. If order doesn't matter, find the number of ways that 2016 can be written as the product of 3 positive integers. For example, one product would be 12*12*14 = 12*14*12 = 14*12*12.

    A. 32 B. 54 C. 64 D. 72 E. 128

20. Let the set S be the empty set. Add the elements of {1, 2} to S, and after 1 minute randomly remove 1 element of S. Next add the 4 elements of {3, 4, 5, 6} to S and after 30 sec randomly remove 2 of the 5 elements of S. Continue this process forever, each time doubling the number of elements added and removed and halving the time. To the nearest hundredth of a minute, find the average length of time 1 remains in S.

    A. 1.30 B. 1.32 C. 1.34 D. 1.36 E. 1.38

Test #2 **AMATYC Student Mathematics League** February/March 2016

1. D
2. C
3. C
4. D
5. B
6. C
7. E
8. B
9. C
10. C
11. 11
12. A
13. D
14. C
15. A
16. A
17. B
18. D
19. Correct for all students
20. D