

1. The price of a meal in a restaurant in France is 24€ (euros). In Europe, the tax of 10% and the service charge or tip of 18% is included in the price. If the euro is worth \$1.20, what should the price of the same meal be in the United States (which does not include tax or tip, both of which are calculated on the base price), to the nearest 50¢?
- A. \$21.50 B. \$22.50 C. \$23.50 D. \$24.50 E. \$25.50
2. An arithmetic progression of positive integers with $a_1 = 2$ and $a_n = 47$ has common difference greater than 1. What is the maximum number of terms it could have?
- A. 10 B. 15 C. 16 D. 17 E. 20
3. The lines with equations $3x + y = a$ and $x - 2y = b$ intersect at the point (2, -4). Find $a + b$.
- A. 2 B. 6 C. 10 D. 12 E. 14
4. Al and Ed are each taking 5 sections at the local community college. They have four sections in common. If the average size of Al's sections is 20, and the average size of all 6 sections is 24, how big is the section Ed is taking without Al?
- A. 40 B. 42 C. 44 D. 48 E. 50
5. The sequence a_1, a_2, a_3, a_4 consists solely of single-digit positive integers. If $a_2 = 2a_1 - 1$, $a_3 = a_2/3$, and $a_4 = 12 - 2a_3$, find a_4 .
- A. 3 B. 4 C. 5 D. 6 E. 7
6. Knights always tell the truth and knaves always lie. A knight sits in a circle with 8 other people, each either a knight or knave. Each of the 9 people says, "I'm sitting next to exactly one knave." Find the maximum possible number of knaves.
- A. 1 B. 2 C. 3 D. 4 E. 5
7. In the equation $A \div MA = .TYC$, different letters are replaced by different digits 1 to 9, and identical letters are replaced by identical digits 1 to 9. Find $A + T$.
- A. 5 B. 6 C. 7 D. 8 E. 9
8. If $(x - 1)^2$ is a factor of $P(x) = 2x^5 - 4x^4 + x^3 + ax^2 + bx + c$, find $a - c$.
- A. 2 B. 4 C. 6 D. 8 E. 10
9. The graph of $xy - 6x + 4y = 36$ is symmetric with respect to (p, q). Find $p \cdot q$.
- A. -36 B. -24 C. 12 D. 24 E. 36
10. In the sequence a_1, a_2, \dots, a_7 of positive integers, a_1, a_2, a_3 and a_5, a_6, a_7 each form a geometric progression, a_3, a_4, a_5 form an arithmetic progression, $a_1 = 5$, and $a_7 = 228$. Find a_4 .
- A. 51 B. 53 C. 55 D. 57 E. 59
11. The equation $a^3 + b^3 + c^2 = 2015$ has exactly one solution in positive integers for which $a > b$. Find $a + b + c$ for this solution.
- A. 38 B. 39 C. 40 D. 41 E. 42

12. For a nonconstant function f , $f(x) = kf(1 - x)$ for all real numbers x and some constant $k > 0$. Find k . A. -2 B. -1 C. 0 D. 1 E. 2
13. How many integers from 1000 to 2015 when tripled have no even digits?
A. 80 B. 81 C. 82 D. 83 E. 84
14. An antibiotic is applied to a bacterial colony at 1 pm, 2 pm, 3 pm, etc, killing 256 bacteria each time. Between applications, the colony doubles in size. If the 6 pm application kills the last 256 bacteria, write the number of bacteria present when the 1 pm application is made in the corresponding blank on the answer sheet.
15. The graphs of the equations $2\sqrt{y} - x + 1 = 0$ and $x + y = 25$ intersect at one or more points (a_i, b_i) . Find the sum of $b_i - a_i$ for all such points.
A. 7 B. 11 C. 47 D. 54 E. 58
16. Cao plans a trip from Seattle to San Francisco to Los Angeles to Las Vegas and return. Between Seattle and SF and SF and LA he can go by car, bus, plane, or train, but between LA and LV he can go only by car, bus, or plane. In how many ways can he select his mode of travel so that at least one of the return legs uses the same mode of travel as the corresponding outbound leg?
A. 720 B. 864 C. 1440 D. 2256 E. 2304
17. The sum of the first n positive integers equals both the sum of the 5 consecutive positive integers starting at a and the sum of the 8 consecutive positive integers starting at b . Find $a - b$ for the least such a and b .
A. 16 B. 21 C. 24 D. 60 E. 63
18. Given a fraction a/b in simplest form, you are allowed to replace it with $a/b + 1$ or with $-b/a$. Find the least number of replacements needed to change $7/9$ into 0.
A. 9 B. 10 C. 11 D. 12 E. 13
19. A quadrilateral has consecutive sides of length 10, 4, and 6 (in that order), and one diagonal divides the quadrilateral into two isosceles triangles. If A_1 and A_2 are the smallest and next smallest areas of quadrilaterals which satisfy these conditions, in which interval below does $A_2 - A_1$ lie?
A. $[0, 1]$ B. $[3, 4]$ C. $[7, 8]$ D. $[8, 9]$ E. $[12, 13]$
20. One number is removed from the set $S = \{8, 12, 22, 24, 29\}$. Each remaining number is multiplied by either $1/2$, 1, or 2 to produce a set T . If the sum of the elements of T equals the sum of the elements of S , which number was removed?
A. 8 B. 12 C. 22 D. 24 E. 29

Test #2

AMATYC Student Mathematics League

February/March 2015

1. B
2. C
3. D
4. C
5. D
6. C
7. E
8. A
9. B
10. A
11. B
12. D
13. D
14. 504
15. A
16. C
17. C
18. E
19. A
20. D