1. If P is the price of the item with tax, $1.08P$ would be the price paid after tax. The discount is $1 - P/(1.08P) = 1 - 1/1.08 \approx 7.4\%$ (Answer: A)

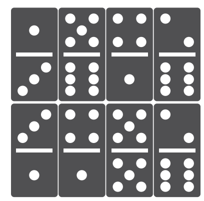
2. Rewrite each line in slope intercept form to find the slope of the first line is $m_1 = -\frac{a}{2}$ and the second is $m_2 = \frac{b}{3}$. If two lines are perpendicular, the product of their slopes is equal to $-1 \implies -\frac{a}{2} \cdot \frac{b}{3} = -1 \implies ab = 6$ (Answer: E)

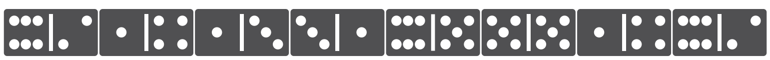
3. Sue's loan decreases by \$200 − \$10 = \$190 each month. $12000/190 \approx 63.2$. So after 63 months, she has paid her loan down to $12000 - 63 \times 190 = 30$. In the 64th month, she pays \$30 plus the \$10 in interest and the loan is paid off. (Answer: E)

4. $3x^2 + 4xy - 4y^2 = (3x - 2y)(x + 2y)$. $\quad 3x - 2y + x + 2y = 4x$ (Answer: A)

5. Solve the system to get the solution $\{(-3, 4)\}$. $a + b = 1$ (Answer: D)

6. The rectangle with the greatest perimeter is formed by placing all 8 dominos end-to-end resulting in a perimeter of 34. The rectangle with the smallest perimeter is a 4 by 4 square, with a perimeter of 16. $34/16 = 2.125$ (Answer: D)





7. The number must have 5 and 9 as a factor. Therefore the last digit must be either a 0 or a 5 and the sum of the digits must be divisible by 9. $(8, 0)$ and $(3, 5)$ both work and both have a sum of 8. (Answer: A)

8. $A_{Ed} + A_{Em} = 28$ and $\frac{1}{2}A_{Ed} = \frac{2}{3}A_{Em}$. Solve the system for just one of the variables and you have the solution. $A_{Ed} = 16$ and $A_{Em} = 12$. Both have 8 oz left. (Answer: C)

9. $2^{60} - 1$ is too big to test in a calculator but $2^{60} - 1 = (2^{30} - 1)(2^{30} + 1)$ and these two numbers are only 10 digits. Using a calculator you can determine $2^{30} - 1$, is divisible by 3, 7, and 13 and $2^{30} + 1$ is divisible by 5 and 11. 17 is the only number in the set that is not a factor. (Answer: D)

10. Let x be the number of 49¢ stamps and y be the number of 3¢ stamps. $0.49x + 0.03y = 4.10$. With $x > y$, there aren't very many combinations. Try the maximum value for x, which is 8 and it works! $x = 8$ and $y = 6$ (Answer: B)

11. Evaluating $\sqrt[4]{2014}$ on your calculator will quickly give you the maximum possible value for a, which is 6. Using the TABLE function, quickly scan for integer values of $Y = \sqrt{2014 - 6^4 - X^2}$. You'll notice, no such values exist for $a = 6, 5, 4$, but for $a = 3$ we get $13 = \sqrt{2014 - 3^4 - 42^2}$ or $3^4 + 13^2 + 42^2 = 2014$ (Answer: B)

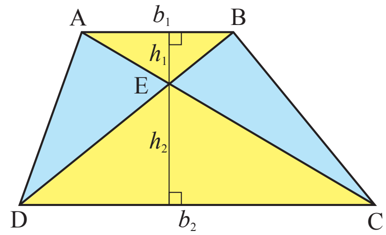
12. This can be done using trial and error. Die 1 = {B, A, C, D, I, N}, Die 2 = {O, P, H, R, X, G}, and Die 3 = {W, Y, T, L, E, S}. The only word that can be spelled is "won." (Answer: E)

13. With a graphing calculator, enter $Y_1 = X/(63 - X)$ and use the table to see if any values round to 0.455. None do, so try $Y_1 = X/(64 - X)$ and it works! The numbers are 20 and 40. (Answer: B)

14. Solve for x in both to get $\dfrac{15 - b}{a} = \dfrac{b - a}{15}$. Now solve for b to get $b = \dfrac{a^2 + 225}{a + 15}$. Use the table on a graphing calculator to find integer values. All possible solutions are for (a, b) are:
$\{(0, 15), (3, 13), (10, 13), (15, 15), (30, 25)\}$. (Answer: C)

15. Add the first equation to -3 times the second equation and to 3 times the third equation to get: $6r + 25s + 28t + 48u + 64v = 37$. (Answer: E)

16. The two triangles are similar and the ratio of their areas is $\frac{75}{48}$ or $\frac{25}{16}$. The ratio of the bases and heights of similar triangles is equal to the square root of the ratio of their areas. If b_1 and h_1 are the base and height of $\triangle$ABE, then $\frac{5}{4}b_1$ and $\frac{5}{4}h_1$ are the base and height of $\triangle$CDE. $A_{\text{trap ABCD}} = \frac{1}{2}(b_1 + \frac{5}{4}b_1)(h_1 + \frac{5}{4}h_1) = \frac{81}{16}(\frac{1}{2}b_1h_1)$. $A_{\triangle\text{ABE}} = \frac{1}{2}b_1h_1 \implies A_{\text{trap ABCD}} = \frac{81}{16}(48) = 243$. (Answer: D)



17. We know $7^3p + 7^2q + 7r + s = 9^3q + 9^2r + 9s + p \implies 680q + 74r + 8s - 342p = 0$. Now we need to find restrictions on the variables to make guess-and-check easier. First, p and q cannot equal zero because the numbers are 4 digits. No digit can be more than 6 because one of the numbers is in base 7. We know q cannot be bigger than 3 because $9^3 \cdot 4 = 2187$ requires five digits in base 7. Suppose $q = 3$, p would have to equal 6 since if $p = 5$ it would not be big enough to bring the equation back to zero. But no combination of r and s work with $p = 3$ and $q = 6$. Similarly, if $p = 2$ we only need to try 4, 5, or 6 for q. The solution is $(q, r, s, p) = (2, 0, 1, 4)$. Which is what we should have tried in the first place since they always try to work the year into these exams! (Answer: 1471)

18. Think of this as distributing 5 chips in 7 bowls. 6 bowls represent the 6 candidates and the 7th represents a vote for no one. If you visualize any given scenario as 5 chips and 6 dividers that separate the chips, the problem is reduced to the number of ways to rearrange 11 things, where 5 are identical and 6 are identical: $\dfrac{11!}{5! \cdot 6!} = 462$ (Answer: C)

19. Counting multiplicity, we know P has 4 roots. If there are only two distinct roots, P can be written $P(x) = (x-a)^2(x-b)^2$ or $P(x) = (x-a)^3(x-b)$. Expanding the first one gives: $P(x) = x^4 - 2(a+b)x^3 + (a^2+b^2+4ab)x^2 - 2ab(a+b)x + a^2b^2$. Solve the system $a^2b^2 = 144$, $-2ab(a+b) = -24$ and you get the roots -4 and 3. So $P(x) = x^4 + 2x^3 - 23x^2 - 24x + 144$. (Answer: D)

20. Start by listing all possible odd-neighbored sets:

n	Sets	Number
	∅	1
1	∅, {1}	2
2	∅, {1}, {1, 2}	3
3	∅, {1}, {3}, {1, 3}, {1, 2, 3}	5
4	∅, {1}, {3}, {1, 3}, {3, 4}, {1, 2, 3}, {1, 3, 4}, {1, 2, 3, 4}	8
5	∅, {1}, {3}, {5}, {1, 3}, {1, 5}, {3, 5}, {1, 2, 3}, {1, 3, 5}, {3, 4, 5}, {1, 2, 3, 5}, {1, 3, 4, 5}, {1, 2, 3, 4, 5}	13

The Fibonacci sequence! When $n = 12$ the total number of sets would be 377, one of which is empty. (Answer: D)