Test #2 **AMATYC Student Mathematics League** **February/March 2012**

1. The probability that the product of the numbers rolled on three fair six-sided dice is prime is A. $\frac{1}{36}$ B. $\frac{1}{24}$ C. $\frac{1}{16}$ D. $\frac{1}{12}$ E. $\frac{1}{8}$

2. If x^2+1 is a factor of $6x^3+5x^2+Px+Q$, then $P+Q=$ A. 10 B. 11 C. 12 D. 13 E. 14

3. Call a date mm/dd/yy *magical* if mm x dd = yy. For example, 12/02/24 is magical, but 02/05/11 and 7/15/05 are not. How many of the following dates can NEVER be magical? January 31 February 29 March 31 April 30

A. 0 B. 1 C. 2 D. 3 E. 4

4. Suppose $a^2 - b^2 = 91$ (a, b integers). If $n = a^2 + b^2 < 1000$, find the units digit of n.

A. 1 B. 3 C. 5 D. 7 E. 9

5. Let S be the set of all lines with equation y = mx + b for which m + b = 36. For how many of the elements of S are both the x- and y-intercepts integers?

A. 8 B. 9 C. 12 D. 15 E. 18

6. A bridge charges 2-axled vehicles a \$5 toll and 3-axled vehicles an \$8 toll. In an hour the bridge collected \$741 from 120 vehicles. If tolls were \$1 higher for 2-axled and \$2 higher for 3-axled vehicles, how much would the bridge have collected?

A. \$888 B. \$908 C. \$926 D. \$934 E. \$1012

7. Let a, b, and c be positive integers which satisfy $a^3+b^3+c^2=2012$. Find $a + b + c$.

A. 28 B. 30 C. 32 D. 34 E. 36

8. Tom, Dick, and Harry each have children. The sum of the number of Tom's children and the average of Dick's and Harry's children is 5, while the sum of the number of Harry's children and the average of Tom's and Dick's children is 7. Find the total number of children in the three families.

A. 7 B. 8 C. 9 D. 10 E. 11

9. In the 5x5 grid at the right, each cell contains one of the digits 1 to 5 so that each row and each column has exactly one of each digit. Find the entry in row 3, column 4.

1	**2**			
2				
				5
			5	**4**

A. 1 B. 2 C. 3 D. 4 E. 5

10. When the 2-digit number aa is multiplied by the 1-digit number $b \neq a$, the result is the 3-digit number cba. Find the sum of all possible values of cba.

A. 264 B. 528 C. 759 D. 891 E. 1045

11. In the equation $x^2 - \frac{10}{9}x + c = 0$, one solution is the square of the other solution. If $c > 0$ is the rational number $\frac{m}{n}$ in simplest terms, find $m + n$.

A. 25 B. 28 C. 32 D. 35 E. 38

12. Which of the following best describes the graph of $(x + y)^2 = x^2 + y^2$?

A. The empty set B. A single point C. two intersecting lines
D. two parallel lines E. A circle

13. Sue and Thai are asked to multiply a positive integer a by a positive integer b and then add a positive integer c to the result. Sue mistakenly first multiplies by c and then adds b, while Thai mistakenly adds b and then multiplies by c. The correct answer was 29, Sue got 59, and Thai got 80. Find $a + b + c$.

A. 12 B. 14 C. 16 D. 18 E. 20

14. Let $A(2, 1)$ and $B(10, 7)$ be points in the xy-plane. Let R be the region in the first quadrant consisting of all points C for which ΔABC has three acute angles. Find the area of R rounded to the nearest integer.

A. 11 B. 61 C. 71 D. 121 E. the area of R is infinite

15. Isosceles ΔABC has base $AB = 4$ and altitude $CP = 6$. Choose point D with $\overleftrightarrow{AD} \perp \overleftrightarrow{AB}$, $AD = AB$, and $\overline{BD}$ intersecting $\overline{AC}$. Choose point E so that $\Delta ADE \cong \Delta ABC$ and $\overline{AE}$ intersects $\overline{BC}$. Find the area common to the two triangles.

A. 7 B. 7.2 C. 7.6 D. 8.0 E. 8.4

16. How many positive integers not divisible by 6 have a base 6 representation which is the reverse of their base 9 representation?

A. 5 B. 6 C. 7 D. 8 E. more than 8

17. Let S be the set of all ordered triples (p, q, r) of prime numbers for which the equation $px^2 + qx + r = 0$ has at least one rational solution. How many primes appear in S at least seven times?

A. 0 B. 1 C. 2 D. 3 E. An infinite number

18. If $\cos\theta = \dfrac{7}{18}$ and $0° < \theta < 90°$, the value of $\tan\dfrac{1}{4}\theta$ can be expressed as $\dfrac{\sqrt{m}}{n}$ where m has no perfect square factors > 1. Find $m + n$.

A. 20 B. 22 C. 24 D. 26 E. 28

19. The Robotics Club has 12 members, 3 each who are freshmen, sophomores, juniors, and seniors. In each of the freshman and junior classes, 1 member is an engineer and two are in CS; in each of the sophomore and senior classes, 2 are engineers and 1 is in CS. Find the number of ways to have a committee of 6 members so that each class and each major is represented on the committee.

A. 492 B. 502 C. 540 D. 592 E. 594

20. The equation $x^2 - 11y^2 + 23 = 10xy$ has four solutions (x_i, y_i) in which both coordinates are integers. Find $|x_1| + |x_2| + |x_3| + |x_4|$.

A. 8 B. 40 C. 44 D. 46 E. 52

1. B
2. B
3. C
4. E
5. E
6. B
7. C
8. C
9. A
10. C
11. D
12. C
13. D
14. Correct for all students
15. B
16. B
17. C
18. B
19. D
20. C