1. (A) $x : 3 \to 7 \to 11$ (steps of 4) and $y : -3 \to 5 \to 13$ (steps of 8)

2. (E) $x(x-1)+(x-1) = x^2-1 = 323 \implies x^2 - 324 = 0 \implies x = \pm 18$ but x must be positive so $x = 18$. $18\Delta 19 = 18 \cdot 19 + 19 = 361$

3. (D) If the perimeter is 36 ft then the sum of the length and width must be 18 ft. Looking half of the rectangle as a right triangle we see $l^2 + w^2 = 170$.
OR: $A = lw = (2lw)/2 = [(w+l)^2 - (w^2+l^2)]/2 = [(P/2)^2 - D^2]/2 = [(36/2)^2 - (\sqrt{170})^2]/2 = (18^2 - 170)/2 = (324-170)/2 = 182 - 85 = 77$.

4. (D) It is simple to verify that the identity is valid for $f(x) = x$ and $f(x) = 2x$, the two sides both being equal to $2x$ and $6x$, respectively. The identity is not valid for $f(x) = \ln x$, however. For example, with $x = 1$, the left side reduces to zero while the right side is undefined.

5. (B) Rewrite the equation as $kx^2 - \sqrt{14}x - 7 = 0$. This has exactly two real solutions when the discriminant is positive: $b^2 - 4ac = 14 - 4(k)(-7) = 14 + 28k > 0$, which is equivalent to $k > -1/2$.

6. (B) Clearly x^{n-2} is a factor of $2009 = 7^2 \cdot 41$, so x is either 1, 7, 41, 49, 287, or 2009. The choices given and the fact $n < x$ require that $x = 7$. Going through the choices, 10 corresponds to $n = 3$, which does not work, but 11 corresponds to $n = 4$ which results in $7^{4-2}(7^2 - 7 - 1) = 49(41) = 2009$.

7. (D) We are given that $\frac{3}{7}W = \frac{1}{2}M$, so the fraction is $\dfrac{\frac{3}{7}W + \frac{1}{2}M}{M+W} = \dfrac{\frac{1}{2}M + \frac{1}{2}M}{M + \frac{7}{6}M} = \dfrac{M}{\frac{13}{6}M} = \dfrac{6}{13}$.

8. (C) In general, if triangle DEF has its vertices on a circle and DE is a diameter, then $\angle F$ is a right angle. It follows that all the corners of the rectangle in this problem are right angles, so $ABCD$ is a rectangle (not necessarily a square).

9. (C) Since each of the center boxes connect to all other boxes but one each, only 1 and 8 can go into these. Once these have been selected, the far left and right boxes are determined, so the middle row can only be either 7182 or 2817. Since one is just a reflection of the other, assume we have 7182 in the middle row. 3 must be in the NW or SW box, and again by symmetry we assume without loss of generality that is is in the NW box. Since 4 and 5 can not be adjacent, they must be in the NE and SW boxes, in either order, and 6 must be in the SE box. Taking into account all possible reflections, $X + Y = 4 + 5 = 5 + 4 = 3 + 6 = 6 + 3 = 9$.

10. (A) Let r be the radius we seek, h the height of the new cone, and H the height of the original cone. By similar triangles, $r/4 = h/H$. We are given that $\dfrac{1}{2} = \dfrac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi 4^2 H} = (r/4)^2(h/H)^2 = (r/4)^3$, therefore $r/4 = \sqrt[3]{1/2} \implies r = 4\sqrt[3]{1/2} = \sqrt[3]{32} = 2\sqrt[3]{4}$.

11. (E) If B is the number of rungs below her at the start, then after 5 more rungs we are told that $2B - 5 = B + 5 \implies B = 10$, so 15 above and below at that point. If there are four times as many below after x more steps, then $15 + x = 4(15 - x) \implies x = 9$.

12. (D) $\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta) = (0.2)(1 + \sin\theta\cos\theta) = (0.2)(1 + \frac{1}{2}\sin 2\theta) = (0.2)(1 + (1/2)(0.96)) = (0.2)(1.48) = 0.296$.

13. (E) There are two vertical asymptotes at $x = \pm 1/100$ and two horizontal asymptotes at $y = \pm 1/100$, so four in all.

14. (A) Complete the square in x to obtain $(x+2)^2 + 2 = y^2 \implies y^2 - (x+2)^2 = 2 \implies (y - x - 2)(y + x + 2) = 2$. If x and y are integers, then the factors on the left are either both even or both odd; either way, their product can not be 2, so there are no integer solutions.

15. Correct for ALL students. (Since the opposite of zero is zero, the problem seems to be that, as worded, the variables are allowed to be zero, so solutions that give different values for $|r| + |s| + |t| + |u|$ are $(r, s, t, u) = (0, 3, 0, 26), (0, 18, 0, 9), (4, 22, 8, -11)$. If the instructions had specified that the variables were all non-zero, then it appears the answer would have been $4 + 22 + 8 + 11 = 45$, so D.)

16. (D) Let $S_n =$ the number of all valid sequences of lights of length n. By drawing a tree of possibilities, we find that $S_1 = 3, S_2 = 6$, and $S_3 = 13$. We could keep drawing, or observe that in general, there are four mutually exclusive possibilities for a valid sequence of n lights: (1) RYX, where X is any valid sequence of length $n-2$, (2) $GRYX$, where X is any valid sequence of length $n-3$, (3) GYX, where X is any valid sequence of length $n-2$, and (4) YX, where X is any valid sequence of length $n-1$. Thus, the total number of valid sequences satisfies $S_n = S_{n-2} + S_{n-3} + S_{n-2} + S_{n-1} = S_{n-1} + 2S_{n-2} + S_{n-3}$. We can use this to find $S_4 = S_3 + 2S_2 + S_1 = 28$ and $S_5 = S_4 + 2S_3 + S_2 = 60$.

17. (B) Divide the second equation by 2 and subtract 6 times the resulting equation from the first equation to obtain the equivalent system $y^3 + 4xy = 0$ and $x^2y + xy = 0$. Since $y \neq 0$, we can divide by y to obtain $y^2 + 4x = 0$ and $x(x+1) = 0$. This last equation implies $x = 0$ or $x = -1$, but $x = 0 \implies y = 0$ because of the other equation, so we must have $x = -1 \implies y^2 - 4 = 0 \implies y = \pm 2$. So there are two solutions, $(-1, 2), (-1, -2)$.

18. (A) Because the equation is symmetric with respect to x and y, we try the change of coordinates $u = (x - y)/2$ and $v = (x + y)/2$. Notice that this sends the line $y = x$ to the line $u = 0$ and the line $y = -x$ to the line $v = 0$. That is, the new coordinates are just a 90 degree rotation of the original coordinates. We have $x = u + v$ and $y = v - u$, so in the new coordinates the equation is $2v = (u + v)^3 + (v - u)^3$ which simplifies to $v = v^3 + 3u^2v$, which is equivalent to $v(3u^2 + v^2 - 1) = 0$. There are two ways this equation can hold: (1) If $v = 0$, which is a line, and (2) if $3u^2 + v^2 = 1$, which is an ellipse.

19. (C) The only way to have the sum of these digits be 16 is if the number is some arrangement of 9511 or 5551. There are $12 + 4$ possibilities in all, so just check these for divisibility by 37 until you find one. $1591 = 37 \cdot 43$, so the answer is $9 + 1 = 10$.

20. (D) Let the rectangle intersect AC in two points F and G, with F between A and G. Then the area in question is $area(ABC) - area(ADF) - area(GEC)$. Using similar triangles, it is easy to find that $|DF| = 16/5$ and $|GE| = 15/8$, so the area of the common region is $\frac{1}{2}[5 \cdot 8 - 2(16/5) - 3(15/8)] = 20 - 16/5 - 45/16 = 1119/80$.