1. Plug in: $f(g(1)) - g(f(1)) = f(2) - g(-1) = 1 - (-2) = 3$. (Answer: E)

2. (2 choices for non-zero first digit) $\cdot$ (3 choices where to place other non-zero digit) $= 6$. (Answer: A)

3. Draw a picture to see the sum is $\arctan(1) + \arctan(1/2) + \arctan(1/3)$. Use a calculator or identity to see this is $90°$. (Answer: C)

4. If x and y are the costs of the two horses, then $200 = .8x \implies x = 250$. Breaking even means $(200 - 250) + .25y = 0 \implies y = 50/.25 = 200$, so $x + y = 450$. (Answer: B)

5. $A(17, 49) = \{17, 18, \ldots, 17 + 49 - 1 = 65\}$ and $A(49, 17) = \{49, 50, \ldots, 49 + 17 - 1 = 65\}$, so 65. (Answer: C)

6. $M = \sum_{n=1}^{k} \ln(an/(bn)) = \sum_{n=1}^{k} \ln(a/b) = k \ln(a/b) = \ln(a/b)^k \implies N = (a/b)^k \implies P = a/b$. (Answer: A)

7. $x = (x^5 \cdot x^5 \cdot x^5)/(x^7 \cdot x^7) = (x^8 \cdot x^8)/(x^5 \cdot x^5 \cdot x^5)$, so I and III both imply that x is rational, but II does not (for example, $x = \sqrt{2}$ is irrational, but $x^6 = 8$ and $x^8 = 16$ are rational). (Answer: B)

8. There are 999 positive integers less than 1000. Of these, $(\frac{1}{3} - \frac{1}{9})(999) = 222$ are divisible by 3 but not 9, and half of these are not even, so the probability is $111/999 = 1/9$. (Answer: B)

9. If r and s are solutions, then $x^2 + 3x + c = (x - r)(x - s) = x^2 - (r + s)x + rs$. Equality of coefficients $\implies r + s = -3$ and $c = rs$. Since $r^2 + s^2 = 33$, we have $9 = (-3)^2 = (r + s)^2 = r^2 + 2rs + s^2 = 33 + 2c \implies c = -12$. (Answer: B)

10. Joe can drop the first ball from height 12. If the first breaks at 12, he can drop the second ball from heights 1, 2, etc. until either it breaks or survives a drop from height 11; if the first does not break at 12, he can redrop it from height $23 = 12 + 11$. If the first breaks at 23, he can drop the second ball from heights 13, 14, etc. until either it breaks or survives a drop from height 22; if the first does not break at 23, he can redrop it from height $33 = 12 + 11 + 10$. And so on. In this way, Joe can determine with certainty if the greatest height is less than $78 = 12 + 11 + \cdots + 2 + 1$. (Answer: E)

11. Let P be the unique point on MT so that $CPTY$ is a square with side length 63. This square has area $63^2 = 3969$, and the right triangle MPC has area $\frac{1}{2}(79 - 63) \cdot 63 = 504$. The hypotenuse of MPC has length $\sqrt{16^2 + 63^2} = 65$, so the remaining triangle MAC is a $3 : 4 : 5$ triangle rescaled by 13, so it is a right triangle with area $\frac{1}{2}(52)(39) = 1014$. The total area is $3969 + 504 + 1014 = 5487$. (Answer: A)

12. Each such set consists of six distinct nonnegative integers. The median is 5, so three numbers are below 5 and three are above (none can equal 5). Therefore, the highest min is 2 and (min,max) = (2,8), (1,9), or (0,10). Since each min/max pair has sum 10 and the average of all six is 5, the other four numbers have sum 20. We can list all sets which meet these conditions: (2, 3, 4, 6, 7, 8), (1, 2, 3, 7, 8, 9), (1, 2, 4, 6, 8, 9), (1, 3, 4, 6, 7, 9), (0, 1, 2, 8, 9, 10), (0, 1, 3, 7, 9, 10), (0, 1, 4, 6, 9, 10), (0, 2, 3, 7, 8, 10), (0, 2, 4, 6, 8, 10), (0, 3, 4, 6, 7, 10). (Answer: A)

13. Since $x^2 + 2x + 2 = (x + 1)^2 + 1 > 0$ for all real x, the equation is equivalent to $x^3 + 4x^2 - 6x - 22 = \pm(x^2 + 2x + 2)$. The $(+)$ equation is equivalent to $x^3 + 3x^2 - 8x - 24 = 0 \implies (x^2 - 8)(x + 3) = 0 \implies x = -3, \sqrt{8}, -\sqrt{8}$. The $(-)$ equation is equivalent to $x^3 + 5x^2 - 4x - 20 = 0 \implies (x^2 - 4)(x + 5) = 0 \implies x = -5, 2, -2$. The sum of the absolute values of these solutions is $3 + 2\sqrt{8} + 5 + 2 \cdot 2 = 12 + 2\sqrt{8} = 12 + 4\sqrt{2}$, so $a + b + c = 12 + 4 + 2 = 18$. (Answer: E)

14. Apart from 999, the digits of such a number add to 9, so it is easy to list them: 117, 135, 153, 171, 315, 333, 351, 513, 531, 711. So there at 11 in all. (Answer: D)

15. Since only the slope matters, we may as well consider $y = 2x$ and $y = -3x$, and $\alpha = \angle AOB$, where $A = (1, 2), O = (0, 0)$, and $B = (-1, 3)$. The y-axis splits α into angles with tangents of 1/2 and 1/3, so $\tan\alpha = (\frac{1}{2} + \frac{1}{3})/(1 - \frac{1}{2} \cdot \frac{1}{3}) = 5/5 = 1$. (Answer: C)

16. For $x \geq 0$, the equation becomes $(y - x)^2 = 0 \iff y = x$ which intersects the unit circle once for $x > 0$. For $x \leq 0$, the equation becomes $y^2 - x^2 = 0 \iff |y| = |x|$ which intersects the unit circle twice for $x < 0$. So there are 3 intersections in all. (Answer: D)

17. $A = (a, f(a))$ and $B = (b, b)$ where $(b - f(a))/(b - a) = -1 \implies b = (a + f(a))/2$. So the distance from A to B is $\sqrt{2((f(a) - a)/2)^2} = \sqrt{2}(f(a) - a)/2$. (Answer: C)

18. The given dimensions imply that SQR is a 45-45-90 triangle and SQP is a 30-60-90 triangle. Since these right triangles share the hypotenuse QS, this hypotenuse is the diameter of a circle which circumscribes the quadrilateral. We have $\angle RTS = \angle QTP = 180 - \angle PQT - \angle QPT = 180 - \angle PQS - \angle QPR = 180 - 60 - (90 - \angle RPS) = 30 + \angle RPS$.

Let C be the center of the circle, so $\angle RCS = 90$ since triangle QRS is isoceles. Since they subtend the same arc, $\angle RPS = \frac{1}{2}\angle RCS = 45$, so $\angle RTS = 30 + 45 = 75$. (Answer: D)

19. Some experimenting makes clear that the non-circumfactorable numbers are those of the form pq, with $p \neq q$ primes. The possibilities for $pq < 200$ with $p < q$ are $2q, 3q, 5q, 7q$, and $11q$, of which there are $24 + 16 + 9 + 5 + 2 = 56$ in all. (Answer: D)

20. The hypotenuse of such a right triangle is a diameter of the circle, so there are $1003 \cdot 2004$ such right triangles (2006/2 choices of diameter, then 2004 choices for the third corner). Since 3 is not a factor of 2006, no such triangles are equilateral, so each such isoceles triangle has a unique vertex, so there are $2006 \cdot 1002$ isoceles triangles (2006 choices of vertex, then $(2006-2)/2$ choices for the base). Since #(right triangles) $= 2 \cdot 1002 \cdot 1003 =$ #(isoceles triangles), $R = I$ and $|R - I| = 0$. (Answer: A)