1. Today 1 euro is worth 1.25 dollars. If the value of the euro in dollars increases by 10% tomorrow, approximately how many euros will 2.20 dollars be worth then?

A. 1.50 B. 1.58 C. 1.60 D. 1.76 E. 1.94

2. The lines with equations 2x - y = a and y - x = b intersect at the point (p, q). Find the value of q.

A. a + b B. a - b C. 2a + b D. a + 2b E. 2a - b

3. Find $\log_{10}\left(\log_{10}\left(\log_{10} 10^{10000000000}\right)\right)$. A. 0 B. 1 C. 2 D. 3 E. 6

4. The digits of a number are rearranged, and the resulting number is added to the original number. How many of the numbers below could NOT equal this sum?
777 7,777 77,777 777,777 7,777,777

A. 0 B. 1 C. 2 D. 3 E. 4

5. Perpendicular lines L and M have equations Ax + By = D and Cx + Ay = E, respectively (A·B ≠ 0). If the sum of these equations is 6x + 10y = 12, one of the lines must have slope

A. -2 B. $-\frac{1}{2}$ C. $-\frac{1}{4}$ D. $\frac{1}{4}$ E. 4

6. In the equation AMA - TYC = SML, identical letters are replaced by the same digit 0 to 9, and different letters are replaced by different digits 0 to 9. If A = 4, which of the following is a possible value of M?

A. 1 B. 3 C. 6 D. 8 E. 9

7. In a sample of 5 positive data values, the median, minimum, and range are all equal, and the mean equals one of the values. The ratio of the maximum to the mean is

A. 1.6 B. 1.75 C. 1.8 D. 2 E. 2.4

8. The points (6, 4) and (2, 10) are symmetric with respect to the line L. An equation for line L is

A. 2x - 3y = 13 B. 3x + 2y = 26 C. 2x + 3y = 29 D. 3y - 2x = 13 E. 2y - 3x = 2

9. The solution to the equation $\left(\log_8 x^2\right)\left(\log_x 8\right)^2 = 1$ satisfies which inequality below?

A. $0 < x \le 1$ B. $1 < x \le 10$ C. $10 < x \le 50$ D. $50 < x \le 100$ E. $x > 100$

10. Knaves always lie; knights always tell the truth. Al says, "Bo is a knight," Bo says, "Cy is a knave," and Cy says, "Exactly one of Al and Bo is a knave". If Al, Bo, and Cy are each either a knight or a knave, it is true that

A. Al and Cy are both knights B. Al and Cy are both knaves
C. Al is a knight, Cy is a knave D. Al is a knave, Cy is a knight
E. it cannot be determined what Al and Cy are

11. The equation $a^4 + 2b^2 + c^2 = 2013$ has a unique solution in positive integers. For this solution, find $a + b + c$. A. 36 B. 37 C. 38 D. 39 E. 40

12. In the sequence $\{a_n\}$, $a_n = \dfrac{a_1 + a_2 + \ldots + a_{n-1}}{n-1}$ for n ≥ 3. If $a_1 + a_2 \neq 0$ and the sum of the first N terms is $12(a_1 + a_2)$, find N. A. 16 B. 18 C. 20 D. 22 E. 24

13. If S = {(*x*, *y*): *x*, *y* are integers and $x^2 = 4y^2 + 81$}, how many elements are in S?

A. 2 B. 4 C. 6 D. 8 E. 10

14. Find the value of *k* for which the equation $\left|k - \left||x| - 6\right|\right| = 2$ has exactly 5 solutions. Write your answer in the corresponding blank on the answer sheet.

15. All fractions $0 < \frac{a}{b} < 1$ (*a*, *b* positive integers) are placed into the sequence $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \cdots$ first by increasing order of denominator and then by increasing order of numerator. Find *a* + *b* for the 2013^{th} element of the sequence.

A. 124 B. 125 C. 126 D. 127 E. 128

16. In parallelogram *ABCD*, $\overline{BC}$ is extended beyond point *C* to point *E*. Points *F* and *G* are the points of intersection of $\overline{AE}$ with $\overline{BD}$ and $\overline{CD}$, respectively. If *FG* = 12 and *EG* = 15, find *AF*.

A. 16 B. 18 C. 20 D. 24 E. 27

17. Ha and Mo play the following game: a fair coin is flipped repeatedly. Ha chooses a 3-outcome sequence, and then Mo chooses a different 3-outcome sequence. Whoever's sequence occurs first wins. If Ha chooses HHH, which choice gives Mo the greatest probability of winning?

A. THH B. THT C. TTH D. HTT E. TTT

18. If Mo chooses the optimal sequence in Problem 17, the probability that Mo wins is

A. $\frac{3}{5}$ B. $\frac{5}{8}$ C. $\frac{3}{4}$ D. $\frac{4}{5}$ E. $\frac{7}{8}$

19. All of the coefficients of the fourth degree polynomial P(x) are odd integers. Find the maximum possible number of rational solutions of the equation P(x) = 0.

A. 0 B. 1 C. 2 D. 3 E. 4

20. In rectangle ABCD, point E lies between A and B and point F lies between B and C. The areas of ΔADE, ΔEBF, and ΔDCF are all equal. If AB = 4 and BC = 2, find the ratio of the area of ΔDEF to the area of ΔADE.

A. $\frac{4\sqrt{3}}{3}$ B. $\sqrt{5}$ C. $2\sqrt{2}$ D. $\frac{2\sqrt{10}}{3}$ E. $\sqrt{6}$

Test #1 AMATYC Student Mathematics League October/November 2013

1. C
2. D
3. B
4. B
5. D
6. B
7. A
8. D
9. D
10. C
11. B
12. E
13. E
14. 8
15. A
16. B
17. A
18. E
19. A
20. B