1. Let x be the length of the side of the square. The perimeter of each of the two rectangles would be $2x + 2(x/2) = 3x = 36 \implies x = 12$. The area of the square is $12^2 = 144$. (Answer: D)

2. Let m_1 and b_1 be last year's prices of milk and bread respectively and m_2 and b_2 be this year's. $m_1 = 1.5b_1$, $m_2 = 1.2m_1$, $b_2 = 1.25b_1$. Now substitute to get an equation of m_2 as it relates to b_2: $m_2 = 1.2m_1 \implies m_2 = 1.2(1.5b_1) \implies m_2 = 1.2(1.5(b_2/1.25)) \implies m_2 = 1.44b_2$. (Answer: C) (Now by induction, all answers to this exam will be 144)

3. Let $a = \angle A$ and $b = \angle B$. Solve the system: $\begin{cases} a = 9b \\ 90 - b = 9(90 - a) \end{cases} = \begin{cases} a = 9b \\ b = 9a - 720 \end{cases}$

(Answer: C)

4. When $x^2 - 10x - 24 = 0 \implies (x - 12)(x + 2) = 0 \implies x = 12$ or -2. (Answer: B)

5. This is the complement of the probability that all three dice show an odd number. The probability that the first die is odd is $\frac{3}{6} = \frac{1}{2}$, the second die is odd is also $\frac{1}{2}$ and the third die as well. So the probability that all three dice show an odd number is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$. Therefore the compliment of this is $1 - \frac{1}{8} = \frac{7}{8}$. (Answer: E)

6. $f(0) = 5 \implies c = 5$, with the other two conditions we get the system: $\begin{cases} a - b + 5 = 10 \\ a + b + 5 = 4 \end{cases}$. Solve to get $a = 2$, $b = -3$, so $f(x) = 2x^2 - 3x + 5 \implies f(2) = 7$. (Answer: A)

7. Consider the rectangle determined by these three points and the vertex $(10, 8)$. The interior of this rectangle contains (10 - 1)(8 - 1) = 63 lattice points. One lattice point will lie on the diagonal at the point (5, 4) because 5 and 4 are the only two integers that have the same 10:8 ratio. So the interior of the triangle will contain half of the 62 remaining points, which is 31. Along the y axis there are 9 more points (8 plus the point at the origin) and 10 additional points on the x-axis. 1 + 31 + 9 + 10 = 51. (Answer: A)

8. Let x and y be the sides of the rectangle. $x + y = 26$ and $x^2 + y2 = 400$. Square the first equation to get $x^2 + 2xy + y^2 = 676$ then substitute for $x^2 + y^2$ from the second equation to get $2xy + 400 = 676 \implies xy = 138$. (Answer: B)

9. Observe the number of single-digit, 2-digit, 3-digit and 4-digit numbers. The first four digits are 2468, then from 10...98 we have 45 2-digit even numbers which represent the next 90 places. Then from 100 to 998 we have 450 3-digit numbers which represent the next 1350 places. So we accounted for $4 + 90 + 1350 = 1444$ places, which leaves $2010 - 1444 = 566$ remaining comprised of 4-digit even numbers starting with 1000. That's 144.5 even numbers which puts us past 1288 by two places, or two places into 1290, or '2'. (Answer: A)

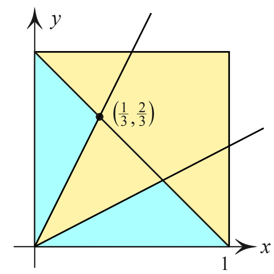
10. The smallest such number will have A = 1 and M = 0. The number must be divisible by 5, so C would be 0 or 5 and since 0 is already taken, C = 5 (not to mention, each choice has 5 as a last digit). Divide 101,235 (the smallest possible number under these constraints) by 35 to get approximately 2892.4. So start with $35 \times 2893 = 101{,}255$ and add 35 repeatedly until you get a number with unique digits (actually we could save time by adding 70 repeatedly because every other number will have a last digit of 0, which is ruled out). We will quickly come up with 101,325. (Answer: A)

11. If the point $(a, \ln(7))$ lies on f then the point $(\ln(7), a)$ lies on f^{-1}. So $\ln\left(a + \sqrt{1 + a^2}\right) = \ln(7) \implies a + \sqrt{1 + a^2} = 7 \implies a = \frac{24}{7}$. (Answer: B)

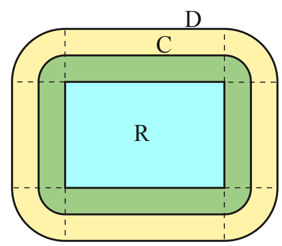
12. $x = \tan\left[\arcsin\left(\frac{13}{14}\right) - 30^\circ\right] = \dfrac{\tan\left(\arcsin\left(\frac{13}{14}\right)\right) - \tan 30^\circ}{1 + \tan\left(\arcsin\left(\frac{13}{14}\right)\right)\tan 30^\circ} = \dfrac{\frac{13\sqrt{3}}{9} - \frac{\sqrt{3}}{3}}{1 + \frac{13\sqrt{3}}{9} \cdot \frac{\sqrt{3}}{3}} = \dfrac{5\sqrt{3}}{11}$. (Answer: A) 1

13. a can be either 1, 2, 3, or 4 because 5^5 is too big. Try $a = 1$. Then we have $b^2 + c^2 = 2009$. If b and c have d as a common factor, we can write them as $b = dx$ and $c = dy$ where x and y are integers. Then $d^2x^2 + d^2y^2 = 2009 \implies x^2 + y^2 = \frac{2009}{d^2}$. For this scenario to work, 2009 must have a perfect square as a factor and when we factor it out the result must be the sum of squares. Indeed, this is the case. That factor is $2009 = 49 \cdot 41$ (so $d = 7$) and $5^2 + 4^2 = 41$. $1^5 + 35^2 + 28^2 = 2010$. (Answer: D)

14. Consider the numbers as ordered pairs on the coordinate axes. All pairs of numbers in $[0, 1]$ would represent the square shown in the figure. All pairs of numbers whose sum is less than 1 can can be expressed as $x + y < 1$. Finally all pairs of numbers with one number being at least twice the other would be represented as $y > 2x$ and $x > 2y$. The area shaded in blue in the figure represents the numbers that satisfy these conditions. Calculate the area of these triangles and divide by the area of the square. (Answer: C)



15. The resulting curve C is a 6 by 8 rectangle with rounded corners of radius 1 (shown in green in the figure). The curve D is an 8 by 10 rectangle with rounded corners of radius 2 (shown in yellow). The area is made up of four quarter-circles of radius 2, $(2^2 \cdot \pi = 4\pi)$, the original 4 by 6 rectangle $(4 \times 6 = 24)$, 2 rectangles that are 2 by 6 $(2 \times 2 \times 6 = 24)$ and 2 rectangles that are 2 by 4 $(2 \times 2 \times 4 = 16)$ for a total area of $64 + 4\pi \approx 76.566$. (Answer: D)



16. The domain of f is $-1 \le x \le 1$ but the range is $y < -1$ or $y > 1$. Therefore numbers that come out of f cannot go back into f. $f(f(x))$ does not exist for any values of x. (Answer: E)

17. $a_n = p + nr + qr^n$, assume the sequence starts at $n = 0$. $a_1 - a_0 = r + qr = 19$ and $a_2 - a_1 = r + qr^2 - qr = 64$. From the second expression you can quickly rule out 12 and 16. A little trial and error will show $r = 4$. The sequence is $a_n = 2 + 4n + 5 \cdot 4^n$. (Answer: B)

18. The first 7 terms are $p, q, pq, pq^2, p^2q^3, p^3q^5, p^5q^8$. So $p^5q^8 = 12{,}500{,}000$. Take the 8th root of 12,500,000 to see that q can be at most 7. Working backwards from 7, you'll find $q = 5$ and therefore $p = 2$. The eighth term divided by the seventh term is the sixth term. $2^3 \cdot 5^5 = 25000$. (Answer: E)

19. (Answer: E)

20. (Answer: B)