1. The solutions to $x^2 - 5x - 6 = 0$, are -1 and 6; the solutions to $x^2 + 4x + 3 = 0$ are -3 and -1. -1 is a solution to both, -3 and 6 aren't. (Answer: E)

2. True for any four consecutive integers. (Answer: D)

3. $x(\frac{1}{x} + b) = y \Rightarrow x = \dfrac{y-1}{b}$ (Answer: A)

4. $(x-2)^2 - (x-2) + 2 = 22 \Rightarrow x^2 - 5x - 14 = 0 \Rightarrow x = -2, 7$ (Answer: E)

5. $r_s = 2.5r_j$, $r_j + r_s = 42 \Rightarrow r_j + 2.5r_j = 42 \Rightarrow r_j = 12$, $r_s = 30$; $0.5(30) + 1.5(12) = 33$ (Answer: C)

6. $4^4 + 9^3 + 32^2 = 2009$ (Answer: D)

7. Digits (permutations): 0-2-1 (4), 0-4-2 (4), 0-6-3 (4), 0-8-4 (4), X-X-X (9), 1-3-2 (6), 1-5-3 (6), etc. (Answer: E)

8. (C) $60 = 1 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \Rightarrow 1 \times 1 \times 60$, $1 \times 2 \times 30$, $1 \times 3 \times 20$, $1 \times 4 \times 15$, $1 \times 5 \times 12$, $1 \times 6 \times 10$, $2 \times 2 \times 15$, $2 \times 3 \times 10$, $2 \times 5 \times 6$, $3 \times 4 \times 5$ (Answer: C)

9. $\dfrac{2\sin x}{\cos x - \sin x \tan x} = \dfrac{2\sin x}{\cos x - \frac{\sin^2 x}{\cos x}} = \dfrac{2\sin x \cos x}{\cos^2 x - \sin^2 x} = \dfrac{\sin 2x}{\cos 2x} = \tan 2x$ (Answer: A)

10. $\Rightarrow xy + 1 = 12y$ and $xy + 1 = \frac{3}{8}x$, subtract the equations to get $y = \frac{1}{32}x \Rightarrow x = 4, 8$ and $y = \frac{1}{8}, \frac{1}{4}$ (Answer: D)

11. $\angle$DAB and $\angle$ABC are supplementary $\therefore \overline{\text{DA}} \parallel \overline{\text{BC}}$. Some textbooks define a trapezoid as a quadrilateral with *at least* one pair of parallel sides and others as *exactly* one pair of parallel sides. (Answer: A or E)

12. (B) By the Fundamental Theorem of Algebra, this polynomial can be factored as $P(x) = 2(x - a)(x-b)(x-c)$, where a, b, c are the three roots. Multiplying this out, we have $P(x) = 2[x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc]$. Setting coefficients of like powers equal, we see in particular that $a + b + c = 3$ and $ab + bc + ca = 3/2$. Since $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$, we have $a^2 + b^2 + c^2 = (a+b+c)^2 - 2(ab + bc + ca) = 3^2 - 2(3/2) = 9 - 3 = 6$. (Answer: B)

13. $2^{2\log_2(2^{1/4}2^{1/8}2^{1/16}\dots)} = 2^{\log_2(2^{1/2}2^{1/4}2^{1/8}\dots)} = 2^{1/2}2^{1/4}2^{1/8}\cdots = 2^{1/2+1/4+1/8+\dots} = 2^{\frac{\frac{1}{2}}{1-\frac{1}{2}}} = 2$ (Answer: C)

14. First some labeling. Starting at the lower right corner and going counterclockwise, label the corners of the trapezoid A, B, C, D. Label the center of the circle Z, and draw 4 radii starting from Z and perpendicular to the four sides of the trapezoid at points E on AB, F on BC, G on CD, and H on DA. The squares ZFCG and ZGDH each have area 16, and the congruent right triangles ZHA and ZEA each have area 16, so it remains to find the areas of the conguent right triangles ZEB and ZFB. But these are similar to ZHA and ZEA, so $|BE|/|EZ| = |ZH|/|HA| \implies |BE| = 2$, and each of the triangles ZEB and ZFB has area 4. Therefore, the total area is $2(16) + 2(16) + 2(4) = 72$. (To see why ZEB is similar to ZHA, observe that C and D are right angles, therefore the angles at A and B add to 180°. Since these are bisected by ZA and ZB, respectively, $\angle ZAH + \angle ZBE = 90°$; since these are right triangles, we must have $\angle ZAH = \angle BZE$ and $\angle AZH = \angle ZBE$, so the triangles are similar.) (Answer: A)

15. We make connections one at a time, starting with any computer we like, call it C1. There are 5 choices for what the first connection from C1 will be to; let us pick one and call that computer C2. We cannot connect C2 back to C1, since that would use up both of the connections for these computers and they would not be connected to the rest of the network, so there are 4 choices for what the next connection from C2 will be; pick one and call it C3. By the same reasoning, C3 can not be connected back to C1 or C2, so there are 3 choices for the next computer, call it C4. There are then 2 choices for the next computer, call it C5, and then C5 must connect back to C1 to ensure that C1 has exactly 2 connections. So it looks like 5! ways to do it, right? Wrong. We would have the exact same network if we had made these same connections in reverse, so we divide by 2 to account for this. For example, connecting computers A, B, C, D, E, F in the order $A \to B \to C \to D \to E \to F \to A$ results in the exact same network as connecting in the reverse order, $A \to F \to E \to D \to C \to B \to A$. Our count of 5! treats these as different, so we divide by 2 to account for the fact they are not. (Answer: D)

16. Write the geometric sequence as $a, ar, ar^2, ar^3, \ldots$ and the arithmetic sequence as $b, b+r, b+2r, b+3r, \ldots$. The sum condition gives three equations in three unknowns: $(E1) a+b=7, (E2) ar+b+r=26$, and $(E3) ar^2+b+2r=90$. Eliminating $b, (E2)-(E1) \implies ar-a=19-r$ and $(E3)-(E2) \implies r(ar-a)=64-r$. Substituting the value of $ar-a$ from the first of these into the second, we obtain a quadratic equation in r, $r^2-20r+64=(r-16)(r-4)=0$. Since 16 is not a choice, 4 is the answer. (Or, alternatively, if $r=16$, then $ar-a=19-r \implies a=1/5$, which is not an integer.) (Answer: B)

17. $1 - P(\text{worker stays on one side}) = 1 - (\frac{8}{13} \cdot \frac{7}{12} \cdot \frac{6}{11} + \frac{5}{13} \cdot \frac{4}{12} \cdot \frac{3}{11}) = \frac{10}{13}$ (Answer: D)

18. A little experimenting leads to quick results. The smallest three digit multiples of the given numbers are 124, 111, ... screech. That looks really promising. 111 + 37 = 148, and now we add multiples of 111 and obtain 259, 370, 481 and there it is (as well as 592, 703, 814, and 925). (Answer: B)

19. The additional area consists of four isosceles right triangles. The side of the original square is partitioned into the length of two legs and a hypotenuse of a 45-45-90 triangle. $10 = x + x + x\sqrt{2} \Rightarrow x = 5(2-\sqrt{2})$. x is the length of each leg of the additional four triangles. Therefore the total area is $A = 10^2 + 4 \cdot \frac{1}{2}(5(2-\sqrt{2}))^2 = 100 + 100(3-2\sqrt{2}) \approx 117$ (Answer: A)

20. We have $a^2 + (a+1)^2 + \cdots + (a+99)^2 = (a+100)^2 + (a+101)^2 + \cdots + (a+198)^2$. Subtracting the first 99 terms from each side, pairing up appropriately, and factoring the differences of squares, we have

$$\begin{aligned}
(a+99)^2 &= [(a+100)^2 - a^2] + [(a+101)^2 - (a+1)^2] + \cdots + [(a+198)^2 - (a+98)^2] \\
&= 100(2a+100) + 100(2a+102) + \cdots 100(2a+296) \\
&= 200(a+50+a+51+\cdots+a+148) \\
&= 200(99a + (148+50) \cdot 99/2) \\
&= 200 \cdot 99(a+99).
\end{aligned}$$

Since $a + 99 \neq 0$, we divide both sides by $(a+99)$ to obtain

$$a + 99 = 200 \cdot 99 \implies a = (200-1) \cdot 99 = (200-1)(100-1) = 19701.$$

(In general, when 100 is replaced by N, the solution is $a = (2N-1)(N-1)$.)