1. The solutions to \( x^2 - 5x - 6 = 0 \), are \(-1\) and \(6\); the solutions to \( x^2 + 4x + 3 = 0 \) are \(-3\) and \(-1\). \(-1\) is a solution to both, \(-3\) and \(6\) aren’t. (Answer: E)

2. True for any four consecutive integers. (Answer: D)

3. \( x(\frac{1}{x} + b) = y \Rightarrow x = \frac{y-1}{b} \) (Answer: A)

4. \((x - 2)^2 - (x - 2) + 2 = 22 \Rightarrow x^2 - 5x - 14 = 0 \Rightarrow x = -2, 7 \) (Answer: E)

5. \( r_5 = 2.5r_j, r_j + r_5 = 42 \Rightarrow r_j + 2.5r_j = 42 \Rightarrow r_j = 12, r_5 = 30; 0.5(30) + 1.5(12) = 33 \) (Answer: C)

6. \( 4^4 + 9^3 + 32^2 = 2009 \) (Answer: D)

7. Digits (permutations): 0-2-1 (4), 0-4-2 (4), 0-6-3 (4), 0-8-4 (4), X-X-X (9), 1-3-2 (6), 1-5-3 (6), etc. (Answer: E)

8. \( (C) 60 = 1 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \Rightarrow 1 \times 1 \times 60, 1 \times 2 \times 30, 1 \times 3 \times 20, 1 \times 4 \times 15, 1 \times 5 \times 12, 1 \times 6 \times 10, 2 \times 2 \times 15, 2 \times 3 \times 10, 2 \times 5 \times 6, 3 \times 4 \times 5 \) (Answer: C)

9. \[ \frac{2 \sin x}{\cos x - \sin x \tan x} = \frac{2 \sin x}{\cos x - \sin^2 x} = \frac{2 \sin x}{\cos^2 x - \sin^2 x} = \frac{\sin 2x}{\cos 2x} = \tan 2x \] (Answer: A)

10. \( xy + 1 = 12y \) and \( xy + 1 = \frac{3}{8}x \), subtract the equations to get \( y = \frac{1}{32}x \Rightarrow x = 4, 8 \) and \( y = \frac{1}{8}, \frac{1}{4} \) (Answer: D)

11. \( \angle DAB \) and \( \angle ABC \) are supplementary; \( \overrightarrow{DA} \parallel \overrightarrow{BC} \). Some textbooks define a trapezoid as a quadrilateral with at least one pair of parallel sides and others as exactly one pair of parallel sides. (Answer: A or E)

12. (B) By the Fundamental Theorem of Algebra, this polynomial can be factored as \( P(x) = 2(x - a)(x - b)(x - c) \), where \( a, b, c \) are the roots. Multiplying this out, we have \( P(x) = 2[x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc] \). Setting coefficients of like powers equal, we see in particular that \( a + b + c = 3 \) and \( ab + bc + ca = 3/2 \). Since \( (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \), we have \( a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca) = 3^2 - 2(3/2) = 9 - 3 = 6 \). (Answer: B)

13. \[ 2\log_2(2^{1/4+1/8+1/16}) = 2\log_2(2^{3/2+1/4+1/8}) = 2^{1/2+1/4+1/8+...} = 2^{1 - \frac{1}{2}} = 2 \] (Answer: C)

14. First some labeling. Starting at the lower right corner and going counterclockwise, label the corners of the trapezoid \( A, B, C, D \). Label the center of the circle \( Z \), and draw 4 radii starting from \( Z \) and perpendicular to the four sides of the trapezoid at points \( E \) on \( AB \), \( F \) on \( BC \), \( G \) on \( CD \), and \( H \) on \( DA \). The squares \( ZFCG \) and \( ZGDH \) each have area 16, and the congruent right triangles \( ZHA \) and \( ZEA \) each have area 16, so it remains to find the areas of the congruent right triangles \( ZEB \) and \( ZFB \). But these are similar to \( ZHA \) and \( ZEA \), so \( |BE|/|EZ| = |ZH|/|HA| \Rightarrow |BE| = 2 \), and each of the triangles \( ZEB \) and \( ZFB \) has area 4. Therefore, the total area is \( 2(16) + 2(16) + 2(4) = 72 \). (To see why \( ZEB \) is similar to \( ZHA \), observe that \( C \) and \( D \) are right angles, therefore the angles at \( A \) and \( B \) add to 180°. Since these are bisected by \( ZA \) and \( ZB \), respectively, \( \angle ZAH = \angle ZBE = 90° \); since these are right triangles, we must have \( \angle ZAH = \angle BZE \) and \( \angle AZH = \angle ZBE \), so the triangles are similar.) (Answer: A)

15. We make connections one at a time, starting with any computer we like, call it \( C1 \). There are 5 choices for what the first connection from \( C1 \) will be to; let us pick one and call that computer \( C2 \). We cannot connect \( C2 \) back to \( C1 \), since that would use up both of the connections for these computers and they would not be connected to the rest of the network, so there are 4 choices for what the next connection
19. The additional area consists of four isosceles right triangles. The side of the original square is 10 ft. Therefore the total area is $A = 10^2 + 4 \cdot \frac{1}{2}(5(2 - \sqrt{2}))^2 = 100 + 100(3 - 2\sqrt{2}) \approx 117$ (Answer: A)

16. Write the geometric sequence as $a, ar, ar^2, ar^3, \ldots$ and the arithmetic sequence as $b, b + r, b + 2r, b + 3r, \ldots$. The sum condition gives three equations in three unknowns: $(E1) a + b = 7, (E2) ar + b + r = 26,$ and $(E3) ar^2 + b + 2r = 90.$ Eliminating $b$, $(E2) - (E1) \implies ar - a = 19 - r$ and $(E3) - (E2) \implies r(ar - a) = 64 - r$. Substituting the value of $ar - a$ from the first of these into the second, we obtain a quadratic equation in $r$, $r^2 - 20r + 64 = (r - 16)(r - 4) = 0.$ Since 16 is not a choice, 4 is the answer. (Or, alternatively, if $r = 16$, then $ar - a = 19 - r \implies a = 1/5$, which is not an integer.) (Answer: B)

17. $1 - P(\text{worker stays on one side}) = 1 - (\frac{8}{13} \cdot \frac{7}{12} \cdot \frac{6}{11} + \frac{5}{13} \cdot \frac{4}{12} + \frac{3}{11}) = \frac{10}{13}$ (Answer: D)

18. A little experimenting leads to quick results. The smallest three digit multiples of the given numbers are 124, 111, ... screech. That looks really promising. $111 + 37 = 148$, and now we add multiples of 111 and obtain 259, 370, 481 and there it is (as well as 592, 703, 814, and 925). (Answer: B)

19. The additional area consists of four isosceles right triangles. The side of the original square is partitioned into the length of two legs and a hypotenuse of a 45-45-90 triangle. $10 = x + x + x\sqrt{2} \Rightarrow x = 5(2 - \sqrt{2})$. $x$ is the length of each leg of the additional four triangles. Therefore the total area is $A = 10^2 + 4 \cdot \frac{1}{2}(5(2 - \sqrt{2}))^2 = 100 + 100(3 - 2\sqrt{2}) \approx 117$ (Answer: A)

20. We have $a^2 + (a + 1)^2 + \cdots + (a + 99)^2 = (a + 100)^2 + (a + 101)^2 + \cdots + (a + 198)^2$. Subtracting the first 99 terms from each side, pairing up appropriately, and factoring the differences of squares, we have

\[
(a + 99)^2 = [(a + 100)^2 - a^2] + [(a + 101)^2 - (a + 1)^2] + \cdots + [(a + 198)^2 - (a + 98)^2]
= 100(2a + 100) + 100(2a + 102) + \cdots + 100(2a + 296)
= 200(a + 50 + a + 51 + \cdots + a + 148)
= 200(99a + (148 + 50) \cdot 99/2)
= 200 \cdot 99(a + 99).
\]

Since $a + 99 \neq 0$, we divide both sides by $(a + 99)$ to obtain

\[
a + 99 = 200 \cdot 99 \implies a = (200 - 1) \cdot 99 = (200 - 1)(100 - 1) = 19701.
\]

(In general, when 100 is replaced by $N$, the solution is $a = (2N - 1)(N - 1)$.)