1. Plug in: $f(-6) - 2f(-3) = (-6)^2 - 2(-3)^2 = 36 - 2 \cdot 9 = 18$. (Answer: E)

2. Only the y values were made negative, so if the original points are on $y = -2x + c$, the new points are on $-y = -2x + c \iff y = 2x - c$, so they are on a line of slope 2. (Answer: D)

3. An 11^{th} degree polynomial has at most 11 x-intercepts and, because 11 is odd, at least 1 x-intercept, so $m = 11$ and $n = 1$ and $m + n = 12$. (Answer: D)

4. In each minute that goes by, Jan cleans 1/20 of the kitchen, Ken cleans 1/12 of the kitchen, and Ben "cleans" $-1/10$ of the kitchen. Working together, they clean $\frac{1}{20} + \frac{1}{12} - \frac{1}{10} = \frac{1}{30}$ of the kitchen every minute, so it takes them 30 minutes to clean the entire kitchen. (Answer: D)

5. If you are not already familiar with these graphs, it is easiest to plot them on a graphing calculator. In the standard window, two intersections are easily seen, one at approximately $x = -0.86$ and another at approximately $x = 1.24$. For $x > 1.24$, the graph $y = x^4$ is above $y = 2^x$, but we know that exponential functions grow faster than polynomials, so there must be a third intersection at some larger value of x; indeed the graphs cross again when $x = 16$, so there are 3 points of intersection in all. (Answer: D)

6. The region is a square of side length 6 with a quarter circle of radius 6 removed, so $A = 6^2 - \frac{1}{4}\pi(6^2) = 36 - 9\pi$. (Answer: B)

7. Let M, S, F be my age now, my son's age now, and my father's age now. We are given $F = 5S, S + (F - M) = M + 8$, and $M + F = 100$. Substituting $F = 5S$ into the other equations and simplifying we obtain $3S - M = 4$ and $5S + M = 100$. Adding these equations we obtain $8S = 104$, so $S = 13$, $M = 3S - 4 = 35$, and I am $M - S = 22$ years older than my son. (Answer: D)

8. The ratio of thefts per person is $(Ae^{at})/(Be^{bt}) = (A/B)e^{(a-b)t}$, where A, B, a, b are constants. Depending on whether $a - b$ is positive, negative, or zero, this could be exponential growth, exponential decay, or constant, but it can not be non-constant linear. (Answer: B)

9. $a^2 - b^2 = (a - b)(a + b)$ and $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, so $a - b$ is a factor of both $33 = 3 \cdot 11$ and $817 = 19 \cdot 43$. Since they have no common prime factors, the only possibility is $a - b = 1$. [Alternative: The difference between consecutive squares is $(n + 1)^2 - n^2 = 2n + 1$, so make a list of the squares of numbers up to $n + 1$, where $2n + 1 = 33 \implies n = 16$. In fact, $a = 17$ and $b = 16$ satisfy both equations, so $a - b = 17 - 16 = 1$.] (Answer: A)

10. The law of cosines says $c^2 = a^2 + b^2 - 2ab\cos(C) \implies \cos(C) = (a^2 + b^2 - c^2)/(2ab)$. Applying this to the given triangle for $C = S, M, L$ and adding the results, we have

$$\cos S + \cos M + \cos L = \frac{7^2 + 8^2 - 6^2}{2 \cdot 7 \cdot 8} + \frac{8^2 + 6^2 - 7^2}{2 \cdot 8 \cdot 6} + \frac{7^2 + 6^2 - 8^2}{2 \cdot 7 \cdot 6} = \frac{47}{32}.$$

(Answer: B)

11. $\cos x = \cot x \cos x \iff \cos x = 0$ or $\cot x = 1 \iff \cos x = \sin x$. The solutions to these two equations with $0 \leq x \leq 2\pi$ are $x = \frac{\pi}{2}, \frac{3\pi}{2}$ and $x = \frac{\pi}{4}, \frac{5\pi}{4}$. The sum of all these solutions is $7\pi/2 = 3.5\pi$. (Answer: C)

12. List the possibilities (tree diagram) using the facts that only A or Y or C can follow A, only T or Y or C can follow M, etc. The possibilities are AYACMT, AYACTM, AYMTCA, ACAYMT, ACTMYA, AAYMTC, AAYMCT, AACTMY, so 8 in all. (Answer: D)

13. $\log_a(\log_b(\log_c x)) = 0 \iff \log_b(\log_c x) = 1 \iff \log_c x = b \iff x = c^b$, so $x_1, \ldots, x_6$ are all the possibilities for c^b, where c, b are any two of the values $2, 4, 8$, so
$N = \log_2(2^N) = \log_2(x_1 \cdots x_6) = \log_2(2^4 2^8 4^2 4^8 8^2 8^4) = 4 \cdot 1 + 8 \cdot 1 + 2 \cdot 2 + 8 \cdot 2 + 2 \cdot 3 + 4 \cdot 3 = 50$.
(Answer: E)

14. The intersection is a right triangle with base 3 and height $3 \tan \theta$, where θ is the angle of overlap: $\angle CAB + (\angle CAB - \theta) = \frac{\pi}{2} \implies \theta = 2\angle CAB - \frac{\pi}{2} = 2 \arctan(4/3) - \frac{\pi}{2}$.
So the area of intersection is $\frac{1}{2} \cdot 3 \cdot 3 \tan[2 \arctan(4/3) - \frac{\pi}{2}] = 1.3125 = \frac{21}{16}$. (Answer: A)

15. In scientific notation, a number $N > 1$ with D digits and leading digit L is equal to $(L + \varepsilon) \times 10^{D-1}$, where $0 \leq \varepsilon < 1$ is the decimal part. Taking $\log_{10}$ we have $\log_{10} N = \log_{10}(L + \varepsilon) + D - 1$. Since $N = 2005^{2005}$, we have $\log_{10}(N) = 2005 \log_{10}(2005) \approx 6620.7393$. From this we determine that $D - 1 = 6620 \implies D = 6621$ and $\log_{10}(L + \varepsilon) = .7393 \implies L + \varepsilon = 5.4869$, so $L = 5$. Therefore, $D + L = 6626$. (Answer: D)

16. All four identities are true. (1) Solve for z in the given: $\cos t + z^2 \cos t = 1 - z^2 \implies z^2 = (1 - \cos t)/(1 + \cos t)$, then take square roots. (2) $\sin^2 t = 1 - \cos^2 t = [(1+z^2)^2 - (1-z^2)^2]/(1+z^2)^2 = [2z/(1+z^2)]^2$, then take square roots. (3) Use $\tan t = \sin t / \cos t$, the previous identity, and the given. (4) $\tan(2u) = \sin(2u)/\cos(2u) = 2 \sin u \cos u/(\cos^2 u - \sin^2 u) = 2 \tan u/(1 - \tan^2 u)$, so set $u = t/2$ to obtain $\tan t = 2 \tan(t/2)/(1 - \tan^2(t/2))$ and compare with the previous identity. (Alternatively, choose a random value for z between 0 and 1, use the given to find t, and check all four identities with a calculator.) (Answer: E)

17. If you are using a TI calculator, type 2, ENTER, 12/(2 * Ans + 5), and then ENTER a whole bunch of times. You will quickly see the numbers converge to 1.5. Alternatively, assume N is large enough that $a_N \approx a_{N+1} \approx L$, where L is the limit these numbers converge to. Then $L \approx a_{N+1} = 12/(2a_N + 5) \approx 12/(2L + 5)$, so we should have $L \approx 12/(2L + 5)$. In fact, this equation is exactly true, and you can solve it to find $2L^2 + 5L - 12 = 0 \implies (2L - 3)(L + 4) = 0 \implies L = 3/2$ or $L = -4$. $L = -4$ doesn't make sense in this problem, so the answer is 3/2. (Answer: A)

18. 25 is odd, so no two points are on a diameter, so no three points form a right triangle, since the right angle would subtend a diameter. So $R = 0$ and we must find $|R - I| = |0 - I| = I$. 3 is not a factor of 25, so no triangle is equilateral, so there are $25 \cdot 12$ isoceles triangles (25 choices for vertex, then $12 = (25-1)/2$ choices for base pairs). The total number of triangles is $C_{25,3} = 25 \cdot 24 \cdot 23/6 = 25 \cdot 23 \cdot 4$, so $I = (25 \cdot 12)/(25 \cdot 23 \cdot 4) = 3/23$. (Answer: D)

19. Add the equations to obtain $x^2 + 2xy + y^2 + 15(x + y) = 54 \implies (x + y)^2 + 15(x + y) - 54 = 0$, which factors as $[(x + y) + 18][(x + y) - 3] = 0$, so $x + y = -18$ or 3. (Answer: A)

20. The angles at A and B are both acute, and the angle at P is right iff it lies on a circle with diameter AB, so P must lie outside this circle to have an acute angle. The probability of this is the area of the square outside the circle, divided by the area of the square $= (1 - \frac{1}{2}\pi(1/2)^2)/1^2 = 1 - \frac{\pi}{8}$. (Answer: D)