

11. Find the sum of all solutions of $\cos x = \cot x \cos x$ for which $0 \leq x \leq 2\pi$.
 A. 1.5π B. 3.25π C. 3.5π D. 3.75π E. 5.5π
12. The letters of AMATYC are rearranged so that the new string starts with A, but no two letters adjacent in AMATYC are adjacent in the new string. How many such strings are there?
 A. 3 B. 5 C. 6 D. 8 E. 9
13. For $i = 1$ to 6, let $\log_a(\log_b(\log_c x_i)) = 0$, where a , b , and c represent every possible different arrangement of 2, 4, and 8. The product $x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5 \cdot x_6$ can be expressed in the form 2^N . Find N .
 A. 19 B. 20 C. 28 D. 33 E. 50
14. A triangle has vertices A(0,0), B(3,0), and C(3,4). If the triangle is rotated counterclockwise around the origin until C lies on the positive y -axis, find the area of the intersection of the region bounded by the original triangle and the region bounded by the rotated triangle.
 A. $\frac{21}{16}$ B. $\frac{25}{16}$ C. $\frac{29}{16}$ D. $\frac{35}{16}$ E. $\frac{75}{16}$
15. When written as a decimal number, 2005^{2005} has D digits and leading digit L . Find $D + L$.
 A. 6623 B. 6624 C. 6625 D. 6626 E. 6627
16. If $0 < t < \frac{\pi}{2}$, $0 < z < 1$, and $\cos t = \frac{1 - z^2}{1 + z^2}$, how many of the following are true?

$$z = \sqrt{\frac{1 - \cos t}{1 + \cos t}}; \quad \sin t = \frac{2z}{1 + z^2}; \quad \tan t = \frac{2z}{1 - z^2}; \quad z = \tan \frac{t}{2}$$
 A. 0 B. 1 C. 2 D. 3 E. 4
17. Let $a_1 = 2$ and $a_{n+1} = \frac{12}{2a_n + 5}$ for all $n \geq 1$. Find the value that a_n approaches as n increases without bound.
 A. $\frac{3}{2}$ B. $\frac{2}{3}$ C. 12 D. 6 E. There is no such value
18. A circle contains 25 points chosen so that the arcs between any two adjacent points are equal. Three of these points are chosen at random. Let the probability that the triangle formed is right be R , and the probability that the triangle formed is isosceles be I . Find $|R - I|$.
 A. $\frac{1}{5}$ B. $\frac{3}{17}$ C. $\frac{1}{7}$ D. $\frac{3}{23}$ E. $\frac{3}{25}$
19. If $x^2 + xy + 15x = 12$ and $y^2 + xy + 15y = 42$, which of the following is a possible value of $x + y$?
 A. 3 B. 6 C. 9 D. 18 E. More than one of these
20. A point P is chosen at random inside square ABCD with AB = 1. Find the probability that all of the angles in $\triangle PAB$ are acute.
 A. $1 + \frac{\sqrt{3}}{4}$ B. $1 + \frac{\pi}{2}$ C. $\frac{1 + \pi}{8}$ D. $1 - \frac{\pi}{8}$ E. $\frac{\pi}{4}$

Test #1

AMATYC Student Mathematics League Answers

October/November 2005

1. E
2. D
3. D
4. D
5. D
6. B
7. D
8. B
9. A
10. B
11. C
12. D
13. E
14. A
15. D
16. E
17. A
18. D
19. A
20. D