Test #1

AMATYC Student Mathematics League

October/November 2004

1. What is the slope of a line parallel to the line with equation \(2x - 5y = 10\)?
   A. \(\frac{2}{5}\)  B. \(\frac{-2}{5}\)  C. \(\frac{5}{2}\)  D. \(\frac{-5}{2}\)  E. -2

2. In square ABCD, point E is between A and B, and point F is between B and C. Find the sum of the measures of \(\angle AEF\) and \(\angle EFC\).
   A. 90°  B. 180°  C. 270°  D. 360°  E. not determined

3. The letters of AMATYC are written as follows: Letters appear in increasing order of the number of line segments or arcs used to write them; identical letters do not appear consecutively. What is the required sequence?

4. A newspaper advertises that it sells the Sunday paper for one-third the price of the rest of the week’s papers. If a weekly subscription costs between $2.20 and $2.30, what is the cost of one Sunday paper and one daily paper?
   A. 56¢  B. 81¢  C. 84¢  D. 87¢  E. $1.12

5. If \(h(x) = 2x - 8\), find \(h'(6)\).
   A. -4  B. 1/4  C. 7  D. 11  E. 20

6. A date is called weird if the number of its month and the number of its day have greatest common factor 1. What are the fewest number of weird days in any month?
   A. 9  B. 10  C. 11  D. 14  E. 15

7. Lucia is not yet 80 years old. Each of her sons has as many sons as brothers. The combined number of Lucia’s sons and grandsons equals her age, and her oldest grandson is 29. How old is Lucia? Place your numerical answer in the corresponding answer blank.

8. What is \(\arccsc \left(\frac{5}{4}\right) + \arccsc \left(\frac{5}{4}\right) + \arccot \left(\frac{5}{4}\right) + \arccot \left(\frac{4}{5}\right)\) ?
   A. \(2\pi\)  B. \(\pi\)  C. \(\frac{\pi}{2}\)  D. \(\frac{\pi}{3}\)  E. \(\frac{\pi}{4}\)

9. George bought groceries with a $10 bill. The cost of the groceries had 3 different digits, and the amount of his change had the same 3 digits in a different order. What was the sum of the digits in the cost?
   A. 13  B. 14  C. 15  D. 16  E. cannot be determined

10. Let \(N\) be the smallest number divisible by 33 which is greater than 1,000,000 and whose digits are all 0’s and 1’s. What are \(N\)’s leading four digits?
   A. 1001  B. 1010  C. 1011  D. 1101  E. 1110

11. In a recent competition, each of three teams played each other team once. In the table, GF is “goals for” (the number of goals scored by a team), and GA is “goals against” (the number of goals scored against a team). What was the score of the S vs J game (Give S’s goals first)?

<table>
<thead>
<tr>
<th>Team</th>
<th>Wins</th>
<th>Losses</th>
<th>Ties</th>
<th>GF</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>F</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>J</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

A. 2-0  B. 2-1  C. 3-1  D. 3-2  E. 4-2
12. The song "What a Beautiful Life" has the lyric, "Day 18,253, well, honey, that's fifty years." If the lyric was supposed to be exactly correct, by how many days is it wrong?
   A. 0 or 1   B. 2 to 4   C. 5 to 7   D. 8 to 10   E. 11 to 13

13. Chris traveled 1 hour longer and 2 miles farther than Calvin, but averaged 3 mph slower. If the sum of their times was 4 hours, what was the sum in miles of the distances they traveled?
   A. 5   B. 26   C. 28.5   D. 30.5   E. 46

14. When five identical tables are placed end-to-end as on the left, the ratio of perimeter to area of the resulting shape is 1/2; when they are placed side-by-side as on the right, the ratio of perimeter to area is 3/10. What is the ratio of perimeter to area of one table?


A. 2/3   B. 3/4   C. 4/5   D. 6/5   E. 3/2

15. Find the sum of the x- and y-intercepts of the line with slope \(-\frac{1}{3}\) which is the hypotenuse of a right triangular region in Quadrant I with legs the x- and y-axes and area \(\frac{392}{3}\).
   A. \(\frac{28}{3}\)   B. \(\frac{56}{3}\)   C. 28   D. \(\frac{112}{3}\)   E. \(\frac{168}{3}\)

16. Let \(A = \{0,1,2,3,4,5,6,7,8,9\}\). How many three-element subsets of \(A\) contain at least two consecutive integers?
   A. 32   B. 40   C. 48   D. 56   E. 64

17. If \(x, y,\) and \(z\) are positive integers with \(x + 2y + 2z = 2005\) and \(2x + 2y + z = 2004\), find the smallest possible value of \(x + y + z\).
   A. 999   B. 1000   C. 1001   D. 1002   E. 1003

18. A store has four open checkout stands. In how many ways could six customers line up at the checkout stands?
   A. 210   B. 1296   C. 4096   D. 60480   E. 151200

19. Circle \(O\) has equation \(x^2 + y^2 = 16\). If \(P\) is \((1,0)\), \(Q\) is \((-1,0)\), and \(R\) is any point on circle \(O\), what is the largest possible value of \(PR + QR\)?
   A. 8   B. \(2\sqrt{17}\)   C. \(6\sqrt{2}\)   D. 17/2   E. \(4\sqrt{5}\)

20. Suppose \(f(x) = ax + b\), \(g(x) = bx + a\) (\(a, b\) integers). If \(f(1) = 8\) and \(f(g(50)) - g(f(50)) = 28\), find the product of \(a\) and \(b\).
   A. 5   B. 12   C. 48   D. 182   E. 210
1. A
2. C
3. CTAYAM
4. C
5. C
6. B
7. 64
8. B
9. B
10. D
11. E
12. D
13. D
14. A
15. D
16. E
17. E
18. D
19. B
20. B